



## Global dynamics of the Lotka–Volterra competition–diffusion system with equal amount of total resources, II

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**Abstract** Part II of this series of three papers, we focus on the joint effects of diffusion and spatial concentration on the global dynamics of a classical Lotka–Volterra competition–diffusion system. For comparison purposes, we assume that the two species have identical competition abilities and the same amount of total resources throughout this paper. Part II is devoted to the case that the spatial distribution of resources for one species is heterogeneous while that of the other is homogeneous. Our results imply that in this case not only is the former guaranteed to survive, in fact, it will often wipe out the latter, regardless of initial values. Asymptotic behaviors of the stable steady states are also obtained for various limiting cases of the diffusion rates.

**Mathematics Subject Classification** 35B40 · 35B30 · 35J57 · 92D25

### 1 Introduction

In an attempt to understand the well known phenomenon—“the slower diffuser always prevails” [5], Lou [17] proposed an approach via “weak” competition. The heart of Lou’s approach is a conjecture, based on his remarkable results in [17], which illustrates a strik-

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