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Title: On a Nonlinear Parabolic Equation Arising from Anisotropic Plane Curves Evolution

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Abstract:

We study the asymptotic behavior of a nonlinear parabolic equation arising from anisotropic plane curves evolution. We show that, if the solution has type-one blow-up, then it will converge (after rescaling) to a self-similar solution. If the solution has type-two blow-up, then its profile near the maximum point is also known.

Let $\psi(x) > 0$ be a smooth $2L\pi$-periodic function, where $L > 0$ is a constant, and let $0 < A_1 < 1$, $A_2 > 1$, be two constants such that

$$0 < A_1 \leq \psi(x) \leq \psi(x) + |\psi_x(x)| + |\psi_{xx}(x)| \leq A_2, \quad \forall\, x \in \mathbb{R}.$$

The goal of this paper is to study the asymptotic behavior of the $2L\pi$-periodic solution $U(x,t) > 0$ to the equation

$$\begin{cases} U_t(x,t) = \psi(x)\, U^p(x,t)\,[U_{xx}(x,t) + U(x,t)] & \text{in} \quad [-L\pi, L\pi] \times [0,T) \\ U(x,0) = U_0(x), \quad x \in [-L\pi, L\pi], \end{cases} \tag{1}$$

where the initial condition $U_0(x) > 0$ is a given smooth $2L\pi$-periodic function. Here $p > 0$ is a constant and we focus on the case $p \geq 2$ in this paper. In terms of the blow-up type of the solution $U(x,t)$, the number $p = 2$ is a threshold value of equation (1). When $p \geq 2$ we begin to have type-two blow-up.