Global Dynamics of the Lotka-Volterra Competition-Diffusion System: Diffusion and Spatial Heterogeneity I

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Abstract

In the first part of this series of three papers, we investigate the combined effects of diffusion, spatial variation, and competition ability on the global dynamics of a classical Lotka-Volterra competition-diffusion system. We establish the main results that determine the global asymptotic stability of semitrivial as well as coexistence steady states. Hence a complete understanding of the change in dynamics is obtained immediately. Our results indicate/confirm that, when spatial heterogeneity is included in the model, "diffusion-driven exclusion" could take place when the diffusion rates and competition coefficients of both species are chosen appropriately. © 2016 Wiley Periodicals, Inc.

1 Introduction

It is well-known that interactions between (random) diffusion and spatial heterogeneity can create some very interesting, perhaps even surprising phenomena in population dynamics in mathematical ecology. Among them two seem particularly outstanding.

First, in 1998, Dockery, Hutson, Mischaikow, and Pernarowski [7] showed that, in a heterogeneous environment, given two competing species with different dispersal rates but otherwise identical, the slower one always wipes out its faster counterpart regardless of their initial values; that is, *the slower diffuser always prevails!* To be more precise, the following Lotka-Volterra competition system was considered in [7]:

$$(1.1)\qquad \begin{cases} U_t = d_1 \Delta U + U(m(x) - U - V) & \text{in } \Omega \times \mathbb{R}^+, \\ V_t = d_2 \Delta V + V(m(x) - U - V) & \text{in } \Omega \times \mathbb{R}^+, \\ \partial_\nu U = \partial_\nu V = 0 & \text{on } \partial\Omega \times \mathbb{R}^+, \\ U(x,0) = U_0(x), \quad V(x,0) = V_0(x) & \text{in } \Omega, \end{cases}$$

where $U(x,t)$ and $V(x,t)$ represent the population densities of two competing species at location $x \in \Omega$ and at time $t > 0$, which are therefore assumed to be nonnegative; Ω, the habitat, is a bounded smooth domain in $\mathbb{R}^N$; $d_1, d_2 > 0$ are the

Communications on Pure and Applied Mathematics, Vol. LXIX, 0981–1014 (2016)