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Finite time blowup of the stochastic shadow Gierer-Meinhardt System*

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Abstract

By choosing some special (random) initial data, we prove that with probability 1, stochastic shadow Gierer-Meinhardt system blows up in finite time in the pointwise sense. We also give a (random) upper bound for the blowup time and some estimates about this bound. By increasing the amplitude of initial data, we can get a blowup in any short time with a positive probability.

Keywords: Stochastic shadow Gierer-Meinhardt system, Finite time blowup, Brownian motions, Itô formula.
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1 Introduction

Inspired by the recent work [9] and [10], we study the blow up of the shadow Gierer-Meinhardt system with random migrations with the following form:

$$\begin{cases} \partial_t u = \Delta u - u + \frac{u^p}{\xi^q} & \text{in } \mathcal{O}\times(0,T), \\ \mathrm{d}\xi = \left(-\xi + \frac{\overline{u^r}}{\xi^s}\right)\mathrm{d}t + \xi \mathrm{d}B_t & \text{in } (0,T), \\ \frac{\partial u}{\partial \nu} = 0 & \text{on } \partial\mathcal{O}\times(0,T), \\ u(0) = u_0 & \text{in } \mathcal{O}, \\ \xi(0) = \xi_0, \end{cases} \tag{1.1}$$

where $\xi \mathrm{d}B_t$ can be explained as random migrations and B_t is a one-dimensional standard Brownian motion. Due to the random effects, we need to introduce the sample space Ω and re-define

$$u(t,x,\omega): \mathbb{R}^+\times\mathcal{O}\times\Omega \to \mathbb{R}^+, \quad \xi(t,\omega): \mathbb{R}^+\times\Omega \to \mathbb{R}^+\setminus\{0\}.$$

The motivation for studying Eq. (1.1) can be found in [16], [15] and [8].

We shall study in this paper the blowup problem of Eq. (1.1) under quite general assumptions. When $p \ge r$ and $\frac{p-1}{r} > \frac{2}{n+2}$, we show that with probability 1, Eq. (1.1)

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