Critical exponent for heat system with time-weighted sources in bounded domain *

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Abstract. This paper mainly considers the coupled parabolic system with time-weighted sources in a bounded domain: $u_t = \Delta u + e^{\alpha t} v^p$, $v_t = \Delta v + e^{\beta t} u^q$ in $\Omega \times (0, T)$ with $\alpha, \beta \in \mathbb{R}$ and $p, q > 0$, subject to null Dirichlet boundary value condition. The critical Fujita curve is determined as $(pq)_c = 1 + \frac{\max\{\alpha+\beta p, \beta+\alpha q, 0\}}{\lambda_1}$, where λ_1 is the first eigenvalue of the Laplacian with null Dirichlet boundary condition, and there is no any additional restriction on α, β, p, q. Next, as an extension, an interesting Fujita phenomenon is observed for another coupled system $U_t = \Delta U + mU + V^p$, $V_t = \Delta V + nV + U^q$ in $\Omega \times (0, T)$ with $pq > 1$ that the critical Fujita curve is represented via the *Fujita critical coefficient* $\max\{m, n\} = \lambda_1$, namely, any nontrivial solutions blow up in finite time if and only if $\max\{m, n\} \geq \lambda_1$. As for the techniques used in this paper, it is mentioned that the current studies of critical Fujita curves for coupled systems (especially in critical cases) seem to be heavenly relying upon Jensen's inequality and/or the Kaplan method, for which one has to deal with complicated discussions on the exponents p, q being greater or less than one. Differently, in the framework of this paper, the heat semigroup is introduced to study critical Fujita curves for coupled system problems, where various superlinear and sublinear cases will be treated uniformly by estimates involved. This greatly simplifies the arguments for establishing the Fujita type theorems. Finally, as applications of the framework of the paper, a new and simpler proof is proposed to some previous results of the authors.

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