

# Global existence and stability of solutions of a semilinear heat equation with exponential reaction term

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## Abstract

In this paper we deal with the following Cauchy problem

$$\begin{cases} u_t = \Delta u + e^u, & x \in \mathbb{R}^N, t > 0, \\ u|_{t=0} = \phi, \end{cases}$$

where  $N \geq 3$  and the initial data  $\phi$  belongs to some suitable class of functions  $\mathcal{E}(\mathbb{R}^N)$ . We obtain the existence of global solutions and finite time blow-up solutions under various conditions on  $\phi$  which depends strongly on the space dimension  $N$ . Meanwhile, we also provide more detailed information on the structure of radial steady state solutions, together with their asymptotic expansions. This enable us to get the stability and weakly asymptotic stability properties for radial steady state solutions.

**Keywords:** Cauchy problem; exponent nonlinearity; stability; instability; asymptotic expansions.

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