AN OPTIMIZATION PROBLEM AND ITS APPLICATION IN POPULATION DYNAMICS

Abstract. This paper is concerned with a diffusive Logistic model in population ecology. As observed by Y. Lou, in a spatially heterogeneous environment, this model can always support a total population at equilibrium greater than the total resources. In other words, the ratio of total population at equilibrium over the total resources is always larger than 1. Our goal is to find the supremum of this ratio taken over all possible choices of spatial distributions of resources and the species' dispersal rate. A conjecture proposed by W.-M. Ni is that, in the one-dimensional case, the supremum is 3. We settle this conjecture and then apply this result to study the global dynamics of a heterogeneous Lotka-Voltera competition-diffusion system.

1. Introduction

Over the past few decades, it has been well accepted, by both mathematicians and ecologists, that spatial characteristics play a significant role in population ecology. In an attempt to understand the joint effects of diffusion and spatial heterogeneity in population dynamics, Lou [10] first investigated the following diffusive Logistic equation

$$\begin{cases} u_t = d\Delta u + u(m(x) - u) & \text{in } \Omega \times \mathbb{R}^+, \\ \partial u/\partial \nu = 0 & \text{on } \partial\Omega \times \mathbb{R}^+, \\ u(x,0) \geq 0, u(x,0) \not\equiv 0, & \text{in } \Omega, \end{cases} \tag{1.1}$$

where $u(x,t)$ represents the population density of a species at location x and time t, which is therefore assumed to be non-negative, d is the random dispersal rate of the species which is assumed to be a positive constant, the habitat Ω is a smooth bounded domain in $\mathbb{R}^N$, $\Delta = \sum_{i=1}^N \frac{\partial^2}{\partial x_i^2}$ is the usual Laplace operator and ν is the outward unit normal vector on $\partial\Omega$. We impose the zero-flux boundary condition to ensure that no individual crosses the boundary of the habitat. The function $m(x)$ is the intrinsic growth rate or carrying capacity, which reflects the environment influence on the species u. Throughout this paper, we assume that $m(x)$ satisfies the following condition:

$$\text{(M)} \qquad \mathrm{m(x) \in L^\infty(\Omega),\ m(x) \geq 0 \text{ and } m \not\equiv const\ \text{ on } \bar{\Omega}.}$$

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