FINITE TIME BLOWUP OF THE STOCHASTIC SHADOW GIERER-MEINHARDT SYSTEM

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ABSTRACT. By choosing some special (random) initial data, we prove that with probability 1, the stochastic shadow Gierer-Meinhardt system blows up pointwisely in finite time. We also give a (random) upper bound for the blowup time and some estimates about this bound. By increasing the amplitude of the initial data, we can get the blowup in any short time with positive probability.

Keywords: Stochastic shadow Gierer-Meinhardt system, Finite time blowup, Brownian motions, Itô formula.
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1. INTRODUCTION

Many of the mathematical models that have been proposed for the study of population dynamics, biochemistry, morphogenesis and other fields, take the following form:

$$\begin{cases} \partial_t u = d_1 \Delta u + f(u,v) & \text{in } \mathcal{O} \times (0,T), \\ \tau \partial_t v = d_2 \Delta v + g(u,v) & \text{in } \mathcal{O} \times (0,T), \\ \partial_\nu u = \partial_\nu v = 0 & \text{on } \partial\mathcal{O} \times (0,T), \end{cases} \tag{1.1}$$

where $\Delta = \sum_{i=1}^n \partial_{x_i}^2$ is Laplace operator, $\mathcal{O}$ is a bounded smooth domain in $\mathbb{R}^n$ with unit outward normal vector ν on its boundary $\partial\mathcal{O}$; the two positive constants d_1, d_2 are the diffusion rates of two substances u and v respectively; $\tau > 0$ is the number tuning response rate of v related to the change of u; f, g are both smooth functions referred to as the reaction terms.

As we choose

$$f(u,v) = \frac{u^p}{v^q}, \quad g(u,v) = \frac{u^r}{v^s}, \tag{1.2}$$

with $p > 0, q > 0, r > 0, s \geq 0$ satisfying the condition:

$$0 < \frac{p-1}{r} < \frac{q}{s+1}, \tag{1.3}$$

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