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Title: On a Length-preserving Nonlocal Flow of Convex Closed Plane Curves

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Abstract:

We study a length-preserving, area-increasing, nonlocal flow of convex closed plane curves and show that this flow evolves such curves into round circles in C^∞ norm. The evolution equation has the form

$$\begin{cases} \dfrac{\partial X(u,t)}{\partial t} = \left[k^\alpha - \dfrac{1}{2\pi}\displaystyle\int_{X(\cdot,t)} k^{\alpha+1}\, ds\right]\mathbf{N}, \quad \alpha > 1, \\ X(u,0) = X_0(u), \quad u \in S^1. \end{cases} \tag{1}$$

Here the initial data $X_0(u) \subset \mathbb{R}^2$ is assumed to be a smooth embedded convex closed curve parametrized by the parameter $u \in S^1$ with counterclockwise orientation; k and $\mathbf{N}$ are the curvature and the inward unit normal vector of the evolving curve respectively. The integral in (1) is with respect to arc length parameter.