Finite Time Blow-up of Parabolic Systems with Nonlocal Terms

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ABSTRACT. In this paper, we study the blow-up phenomena for a class of parabolic systems with nonlocal terms, called *shadow systems*, which are often used to approximate reaction-diffusion systems when one of the diffusion rates is large. Existence of finite blow-up solutions is characterized based on the parameters in the shadow systems. Two different approaches are employed to overcome the difficulties caused by the appearance of nonlocal terms and the lack of comparison principles. One is based on integral estimates, while the other relies on the Schauder Fixed-Point Theorem. This paper continues the work begun in [13]. In particular, we improve the earlier results concerning blow-up solutions to the optimal case.

1. INTRODUCTION

Reaction-diffusion systems of the following form have been used extensively in modeling various phenomena in many branches of science:

$$\begin{cases} u_t = d_1 \Delta u + f(u,v) & \text{in } \Omega \times (0,T), \\ \tau v_t = d_2 \Delta v + g(u,v) & \text{in } \Omega \times (0,T), \\ \dfrac{\partial u}{\partial \nu} = \dfrac{\partial v}{\partial \nu} = 0 & \text{on } \partial\Omega \times (0,T), \\ u(x,0) = u_0(x),\ v(x,0) = v_0(x) & \text{in } \Omega, \end{cases} \tag{1.1}$$

where $\Delta = \sum_{i=1}^{n} \partial^2/\partial x_i^2$ is the usual Laplace operator, Ω is a bounded smooth domain in $\mathbb{R}^n$ with unit outward normal vector ν on its boundary $\partial\Omega$, two positive constants d_1 and d_2 represent the diffusion rates of the two substances u, v, respectively, the number $\tau > 0$ is related to the response rate of v versus the change