DYNAMICS AND ASYMPTOTIC PROFILES OF ENDEMIC EQUILIBRIUM FOR TWO FREQUENCY-DEPENDENT SIS EPIDEMIC MODELS WITH CROSS-DIFFUSION*

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ABSTRACT. This paper is concerned with two frequency-dependent SIS epidemic reaction-diffusion models in heterogeneous environment, with a cross-diffusion term modeling the effect that susceptible individuals tend to move away from higher concentration of infected individuals. It is first shown that the corresponding Neumann initial-boundary value problem in an n-dimensional bounded smooth domain possesses a unique global classical solution which is uniformly-in-time bounded regardless of the strength of the cross-diffusion and the spatial dimension n. It is further shown that, even in the presence of cross-diffusion, the models still admit the threshold-type dynamics in terms of the basic reproduction number $\mathcal{R}_0$; that is, the unique disease free equilibrium is globally stable if $\mathcal{R}_0 < 1$, while if $\mathcal{R}_0 > 1$, the disease is uniformly persistent and there is an endemic equilibrium which is globally stable in some special cases. Our results on the asymptotic profiles of endemic equilibrium illustrate that restricting the motility of susceptible population may eliminate the infectious disease entirely for the first model with constant total population but fails for the second model with varying total population. In particular, this implies that such cross-diffusion does not contribute to the elimination of the infectious disease modelled by the second one.

1. INTRODUCTION

In this paper, we are interested in the following two diffusive SIS epidemic models with cross-diffusion and frequency-dependence:

$$\begin{cases} S_t = d_S \Delta S + \chi \nabla \cdot (S \nabla I) - \beta(x) \dfrac{SI}{S+I} + \gamma(x) I, & x \in \Omega, t > 0, \\ I_t = d_I \Delta I + \beta(x) \dfrac{SI}{S+I} - \gamma(x) I, & x \in \Omega, t > 0, \\ \dfrac{\partial S}{\partial \nu} = \dfrac{\partial I}{\partial \nu} = 0, & x \in \partial\Omega, t > 0, \\ (S(x,0), I(x,0)) = (S_0(x), I_0(x)), & x \in \Omega \end{cases} \tag{1.1}$$ SIS-1

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