ON A HYPERBOLIC EQUATION ARISING IN ELECTROSTATIC MEMS

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Abstract

We consider a damped wave equation with singular nonlinearity and Dirichlet boundary condition in a bounded domain. This equation models an electrostatic micro-electro-mechanical system (MEMS) device. We show that the pull-in voltage λ^* is the critical threshold for global existence and quenching in this wave equation. More precisely, if the applied voltage $\lambda < \lambda^*$, then the equation admits a unique global small solution that exponentially converges to the minimal steady state, while large solution may quench in finite time. If $\lambda > \lambda^*$, then any solution quenches in finite time. Finally, we analyze the relation between the hyperbolic model and the parabolic model through the viscosity dominated limit. Based on valid asymptotic expansions, we show that the solution of the hyperbolic equation can be approximated by the solution of the parabolic equation plus an initial layer part. In this singular limit, we also derive the error estimates in arbitrary order.